Decentralized Exchange

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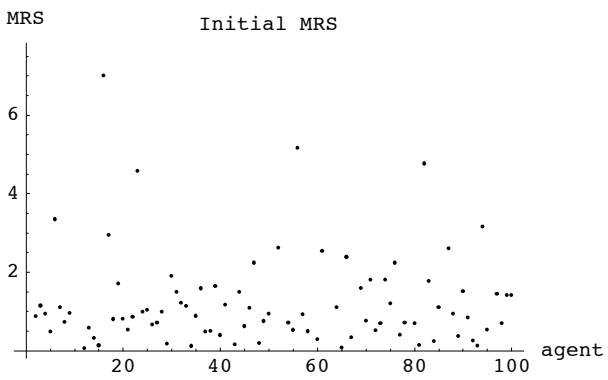
We have a society consisting of *nr = 100* agents,
each of them having a random endowment (between 1 and 100) of two goods. Two agents are drawn randomly from the society and engage in trade. They trade at a price x which is the geometric average of the two marginal rates of substitution. The amount traded is proportional to the parameter size. Agents have a Cobb – Douglas utility function, $u(x, y) = x^{0.5} y^{0.5}$.

We run the simulation with *trades = 2000*. We observe that MRS get closer together (look at mean and variance of MRS before and after the trades and at the two MRS plots).

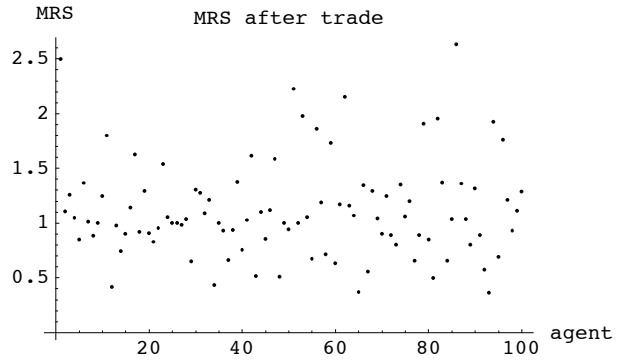
We compute the initial total utility of society, which we define as the unweighted sum of individual utilities. We compare total utility of the society before and after the trades took place, and compare this to the value of total utility that would be reached by trading only at the equilibrium price. (The equil. price is one.)

```
In[61]:= Needs["Statistics`Master`"];
         SeedRandom[133];
         trades = 2000; (* number of trades *)
         size = 1;
         k = 0;
         nr = 100; (* number of agents *)
         averagemrs = {};
         society =
           Table[{++k, {Random[Integer, {1, 100}], Random[Integer, {1, 100}]}}, {nr}];
         initial = society;
         initialrate[agent_] := N[initial[[agent]][[2]][[1]] / initial[[agent]][[2]][[2]]]
         initialrates = Table[initialrate[i], {i, nr}];
         AppendTo[averagemrs, Mean[initialrates]];
         ListPlot[initialrates, PlotLabel → "Initial MRS", AxesLabel → {"agent", "MRS"}];
         mrs[u_, v_] := If[u[[1]] > 0 && u[[2]] > 0 && v[[1]] > 0 && v[[2]] > 0,
           With[{x = N[Sqrt[(u[[1]] v[[1]]) / (u[[2]] v[[2]])]]},
            size {1/(1 + x), -x/(1 + x)} Sign[u[[2]] v[[1]] - u[[1]] v[[2]]]]
           , {0, 0}]
         rate[agent_] := N[society[[agent]][[2]][[1]] / society[[agent]][[2]][[2]]]
         rates := Table[rate[i], {i, nr}]
         util[soc_] := Sum[N[Sqrt[soc[[i]][[2]][[1]]] * Sqrt[soc[[i]][[2]][[2]]]], {i, nr}]
         evol = {};
         AppendTo[evol, util[initial]];
         equtil = Sum[N[(initial[[i]][[2]][[1]] + initial[[i]][[2]][[2]]) / 2], {i, nr}];
```

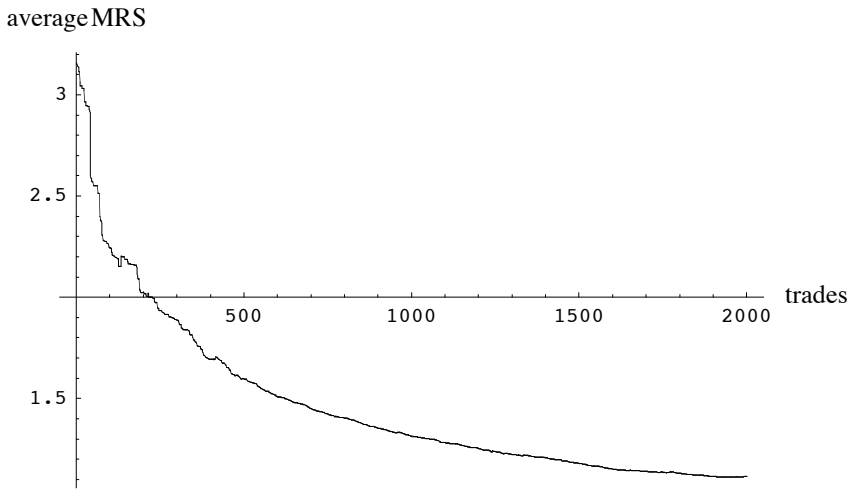


```
In[79]:= Do[
           {r = society[[Random[Integer, {1, nr}]]];
            s = society[[Random[Integer, {1, nr}]]];
            trader1 = r[[1]];
            trader2 = s[[1]];
            u = r[[2]];
            v = s[[2]];
            z = mrs[u, v];
            society = ReplacePart[society, {trader1, u + z}, trader1];
            society = ReplacePart[society, {trader2, v - z}, trader2];
            AppendTo[averagemrs, Mean[rates]]; AppendTo[evol, util[society]]}, {trades}];
```

```
In[80]:= ListPlot[rates, PlotLabel → "MRS after trade", AxesLabel → {"agent", "MRS"}];
```




```
In[81]:= ListPlot[averagemrs, PlotJoined → True, AxesLabel →
           {FontForm["trades", {"Times", 12}], FontForm["average MRS", {"Times", 12}]}];
```




```
In[82]:= Mean[initialrates]
         Variance[initialrates]
         Mean[rates]
         Variance[rates]
```

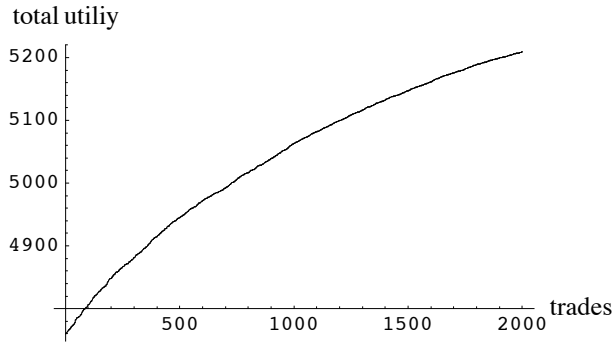
```
Out[82]= 3.15857

Out[83]= 57.4796

Out[84]= 1.11505

Out[85]= 0.19701
```

```
In[86]:= ListPlot[evol, PlotJoined → True,  AxesLabel →
           {FontForm["trades", {"Times", 12}], FontForm["total utiliy", {"Times", 12}]}];
```




```
In[87]:=

In[88]:= util[initial] (* total utility before trading *)
         util[society] (* total utility after trading *)
         equtil (* total utility if trading at equil. prices *)

Out[88]= 4759.05

Out[89]= 5209.03

Out[90]= 5306.5
```

References

Albin, Peter S. and Duncan K. Foley; Cellular Automaton Models of Economic Exchange